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Jocelyn Ireson-Paine*

The Medium Wrecks the Message: Describing Artistic Style Using a Relational View of Art

Abstract

I formulate style as "compensatory triangles," describing how naïve depictions get rebalanced by "pictorial techniques" to compensate for information loss and interference from their medium. I also formulate the information lost, perhaps because of interference, when translating between styles. The formulations may be helpful in teaching. They, and proposals for further mathematisation related to deep-learning style transfer, are inspired by category theory and a relational view of art.

Keywords

Artistic Style, Aesthetic Balance, Influence of Medium, Category Theory, Art as Transformations

Introduction

This paper was inspired by applying a branch of maths called category theory to cartooning. It is an unusual combination, so I must explain why I am doing this.

A terminological point: by "art," I shall mean drawing and painting. However, the ideas extend to media such as film and sculpture, to acting (Walter 1999 pp. 190–191), and non-representational arts such as music (Joncas 2020), dance (Mannone & Turchet 2019), and fashion (Ireson-Paine 2021b, Chapter 22).

* Independent Researcher
Email: popx@j-paine.org

Category theory is one consequence of a late-19th century mathematical revolution wherein mathematicians reformulated geometry to study geometrical transformations (Tao 2013). They asked questions such as what remains unchanged by such transformations and whether there is an "essence" that all objects share even though they differ from one another.

This revolution led mathematicians to focus more on transformations and less on the objects transformed. In category theory, this reaches its pinnacle. I claim we should extend this world-view to art and think of aesthetic transformations (§2). The items transformed will be drawings, meanings, styles, etc. The paper is exploratory, but in other fields, including computer science (Goguen 1991) and the semiotics of metaphor (Joncas 2020), category theory has proven an excellent tool for formalising and organising and driving intuition. So explorations are worthwhile.

Category theory studies "categories." These are mathematical objects whose name has nothing to do with the everyday use of the word. Ignore the latter, or it will be confusing. A category is a network whose nodes are items of interest. They are connected by arrows representing transformations, mappings, comparisons, and functions that relate one item with another.

That is what categories contain, but what is their function? A category is a "workspace" where items and their relations are laid out for mathematical study. Typically, we study the relations. One relation might state that one item is essentially the same as another, despite superficial differences. Another, that one item is part of another and fits inside it. There are many possibilities.

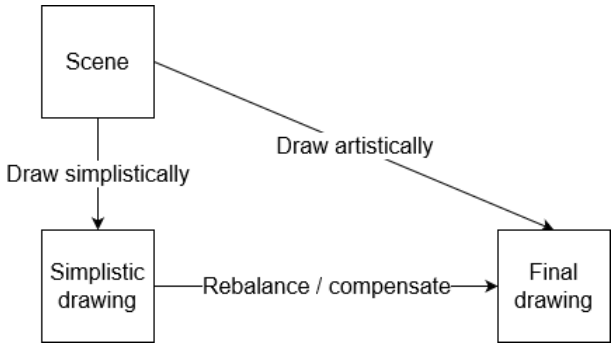
We also study similarities between categories. Often, we ask how one category is "mirrored" in another. Because even though the items in one might be very different from those in the other, the pattern of relations can be the same. It is as if the categories were Perspex sheets which we want to superimpose. So categories help study translations between different conceptual systems, such as art styles.

When I asked the questions that led to this paper, categories were in my mind. I was on one side of Oxford's Cornmarket Street, sketching someone on the other side. How detailed, I wondered, should I make their hands? Should I draw the boundaries of all the fingers, or would that clutter the sketch? If I do not draw them all, aren't I lying? However, how can I justify lying? Furthermore, how big a lie am I allowed to tell if I can? After all, I also program computers, and there, I must not lie at all.

The hands brought to mind a technique many cartoonists use when drawing repeated textures such as bricks, grass, or stones. They draw only texture patches, leaving the viewer's brain to fill in the rest (§3).

Why do this? It saves work, and it can also restore tonal balance. In pen drawings, areas where all the texture is shown, may look too dark and grab too much attention. So removing texture restores what we might call attentional balance.

At this point, category theory made me think of a triangle—I call it a "compensatory triangle"—of transformations (§5):



The long arrow represents the transformation from the original scene to the final drawing. The two shorter arrows say this is the composite of two other transformations. The first draws the original scene simplistic and formulaic, rendering all visible edges as lines. The second restores balance to that drawing by deleting texture. I call this a "rebalancing" or "compensatory" transform.

This figure inspired two mathematical observations. The first was that some transformations have "inverses," i.e., transformations that undo them. Thus, the inverse of "walk east one foot" is "walk west one foot." The rebalancing transform is not an inverse. However, it is close; it is an almost-inverse (§6) which undoes the bad effects of the simplistic-drawing transformation as best it can. It cannot undo them completely, as the original scene and the drawings are in different graphic languages. However, it can try a workaround: reducing darkness by deleting lines rather than lightening or thinning them.

The second observation was that we should catalogue other artistic transformations and ask which counts as rebalancings. This observation led to a database of art techniques (Ireson-Paine 2021a), insights into how artworks behave when scaled and rotated (Ireson-Paine 2021b, Chapter 22), and the examples from §3–§15.

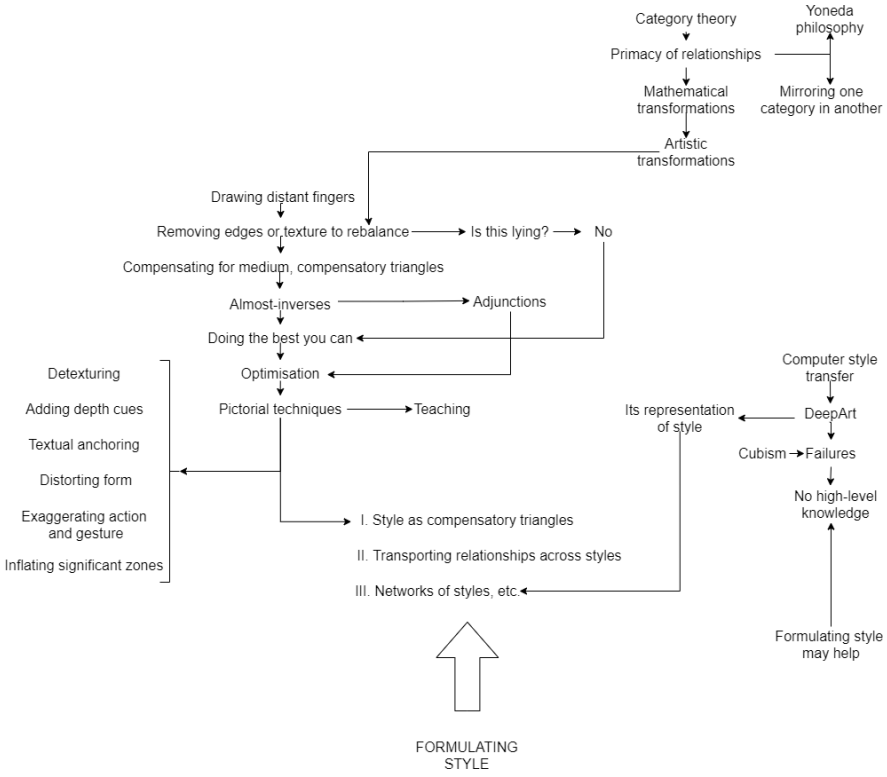
In those sections, I use the phrase "pictorial technique." This phrase came about indirectly. Doing the best one can undo the bad effects means getting as close to a goal (restoring tonal and attentional balance) as possible. Nevertheless, "getting as close as possible" is optimisation. So when I draw, I optimise. However, the almost-inverses mentioned above seemed akin to "adjunction," an actual category-theoretic construction closely related to optimisation (Critch 2009). Searching for other researchers who also regarded art as optimisation led to Frédo Durand and his "pictorial techniques" (§9). By this, Durand means techniques that transform a "direct recording" of the scene to produce the same effect on the viewer as the original. These are like my rebalancing but act on photos or other direct recordings rather than formulaic drawings.

My first formulation of style (§16) arose from these: express it as a collection of compensatory triangles, where each triangle shows how an artist uses a particular pictorial technique.

The second (§18) was inspired by the relational view and mirroring. It considers relations between styles and asks how far can an artwork be done in one style mirror one done in another? When does the medium prevent mirroring?

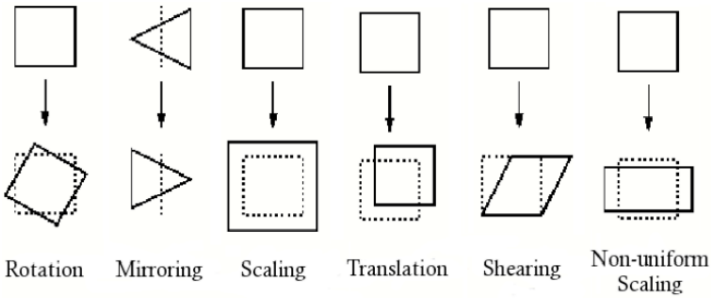
We should explore other directions too. There is a popular method of restyling one artwork in another style, deep-learning style-transfer (§19). I show experimentally (§20) and by argument (§21) that, although impressive, it does not "know" enough about style to avoid mistakes. The precise mathematical formulation might give it that knowledge and be attractive for its own sake. So it is worth exploring further (§22).

The diagram below summarises the main links between ideas herein. In addition, almost-inverses and optimisation contribute to formulation I, "mirroring" to II and the Yoneda philosophy (§22) to III.



1. A relational view of art

I mentioned the shift from focussing on mathematical objects to focussing on their transformations and proposed that we shift our world-view of art in the same way. Here is one example of mathematical transformations to make this concrete, familiar to users of programs such as Photoshop. They are stretches, scalings, rotations, and other transformations of two-dimensional shapes (de Vries 2006).



Artistic transformations include those from §3–§15: detexturing; inserting form cues; textual anchoring; exaggerating action and gesture; inflating regions crucial for recognition. Others are catalogued in (Ireson-Paine 2019; Ireson-Paine 2021b).

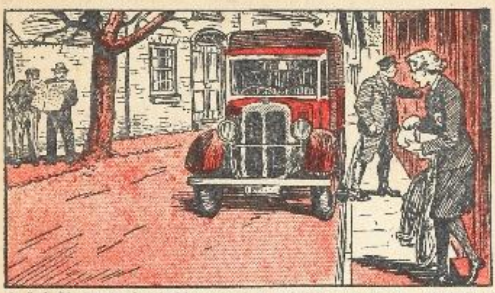
As an analysis analogous to those of (Vandoulakis & Stefaneas 2015; Vandoulakis 2017) would show, I am primarily concerned with transformations that help depict geometry and texture. Future papers will apply these ideas too, for example, depicting emotion in Expressionism.

2. Translate, then delete cues to restore balance

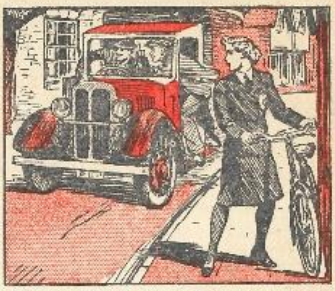
As mentioned, cartoonists often draw only patches of texture. These examples demonstrated with cartoon bricks (Clipart Library n.d.; Private Eye 2019); realistic bricks (Tip Top Book 1953); fur (Hart 2000); and grass, weeds, and cobbles (The Garden Machine Centre 1969):

EXCITING HOLIDAY

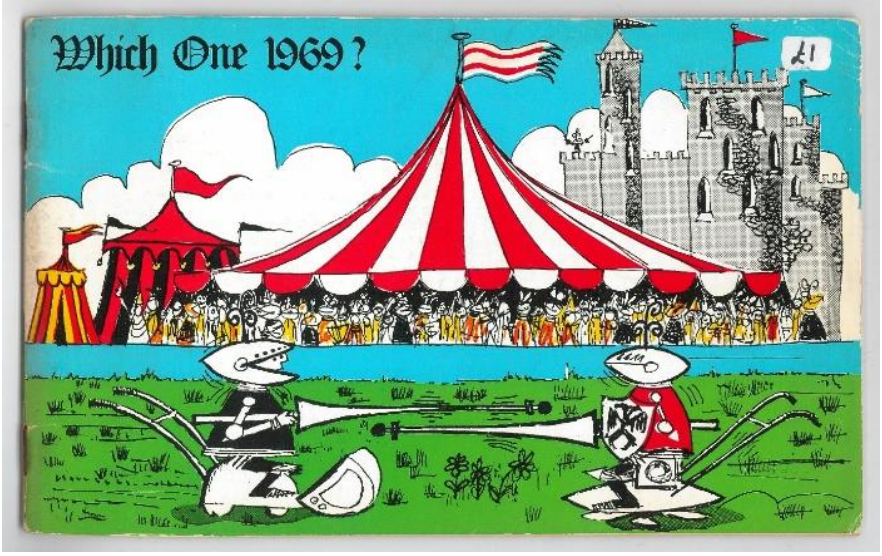
The Romantic Story of a Very Brave Schoolgirl

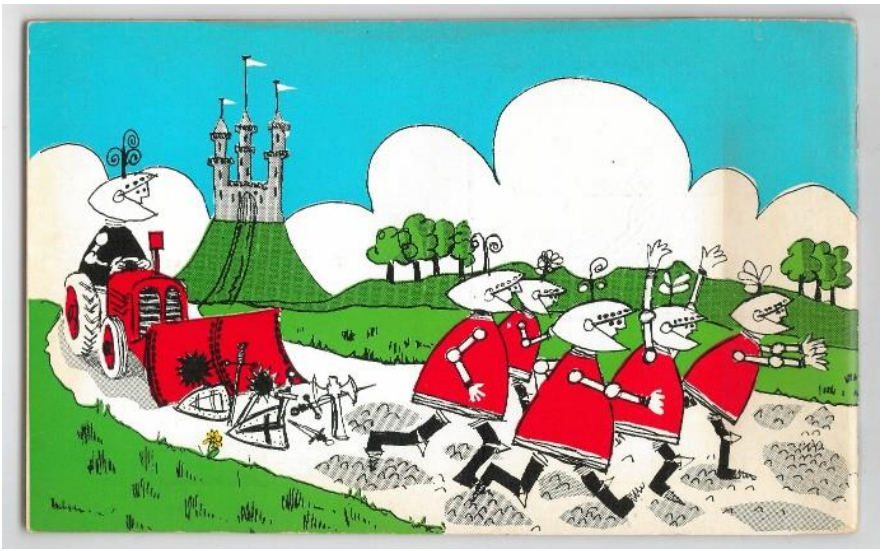


1. Judy Morley, head girl of her school, is on holiday and decides to cycle over to her aunt's lovely house in the country. In the village Judy stops to buy some sweets. As she leaves the shop, she sees a fairly large motor van pulled up nearby.



2. Suddenly, to her surprise and amazement, she sees two men hurry across the road and climb into the driving cab of the van. Judy is certainly puzzled, for she has already seen Mr. Brown, the owner of the van, go into one of the village shops.





Detexturing compensates for the fact that drawing all the texture would pull the viewer's attention away from the rest of the drawing and spoil tonal balance. Imagine how cluttered a drawing of these Oxford houses (Ireson-Paine 2015) would look if it showed every line of mortar:

I think of this as follows. The language of pen and ink is poorer than that of reality because it cannot express subtle differences in tone.

So drawing lines between the bricks makes those lines stand out much more than they do in reality.

To restore balance, we must make them stand out less.

If it were, we could lighten or thin them, but we cannot because our language is not rich enough. So we do the next best thing and remove some of the lines altogether, leaving just enough to suggest that there are bricks.

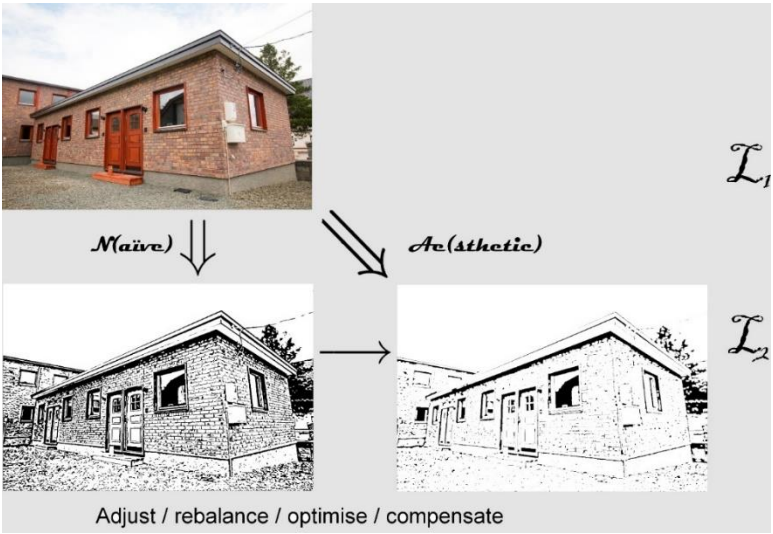
This practice gives an exciting interchange of properties. We want to reduce average darkness by lightening or thinning our lines, but changing that property is not allowed. So we change another one, the number of lines.

3. Detexturing has costs

However, there are costs. The viewer cannot know whether a blank patch is blank because the texture was deleted or there was never any. That is inevitable. Because the rebalancing transform adjusts the "wrong" property, it interferes with information that it should not touch. However, it cannot help touching it. I discuss this in my second formulation of style (§18).

4. Detexturing as a compensatory triangle

Here is detexturing as a diagram. The vertical arrow represents making a simplistic or "naïve" line drawing, and the horizontal arrow detextures it, removing bricks. (I use "naïve" with its everyday meaning, not that of "naïve art.") They combine to give the diagonal arrow. The $\mathscr{L}$s signify different languages: that of reality and that of pen and ink.

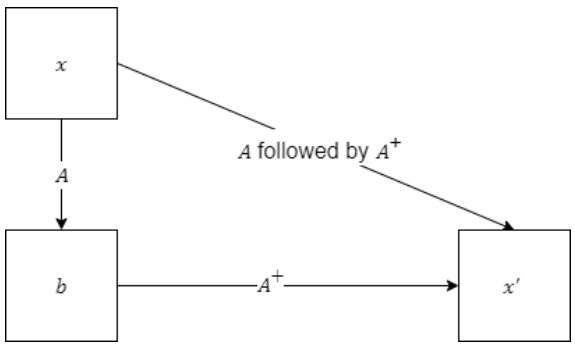


I would have liked the bottom two images to be cartoons. The one on the left would show all the bricks; that on the right would show only those needed to convince the viewer there are bricks and maintain tonal balance. Nevertheless, I did not have time to draw two cartoons, so I have simulated them by greyscaling and thresholding the first image to make the second, then debricking that to make the third.

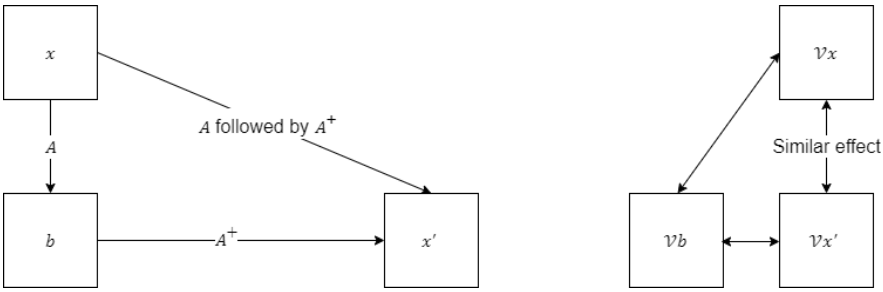
5. Detexturing as a generalised inverse

Mathematically speaking, detexturing resembles a "generalised inverse" (Generalised Inverse 2021). It is an operation that undoes another operation, not precisely, but as closely as circumstances permit. Here, the circumstances change visual language from reality to pen and ink, and the latter allows only a few "moves." No combination thereof can completely undo the damage wreaked by translation. So we look for moves that do the best they can.

We can express this in the notation from (Dataplot 2009). Let x be a scene, and A transform it to a simplistic drawing b. Symbolically, $b=Ax$. Let A^+ be the rebalancing transform. Then A^+b partially reverses A. It undoes its unwanted effects and finds an x' that is as close to x as possible while acknowledging that x and x' can never be identical:



For the word "close" to make sense, some notion of distance between images is needed. One possibility is to compare the effects on the viewer (§8). This comparison can be expressed as below, where we augment the diagram with three boxes on the right, each representing the viewer V's response to scene or drawing:

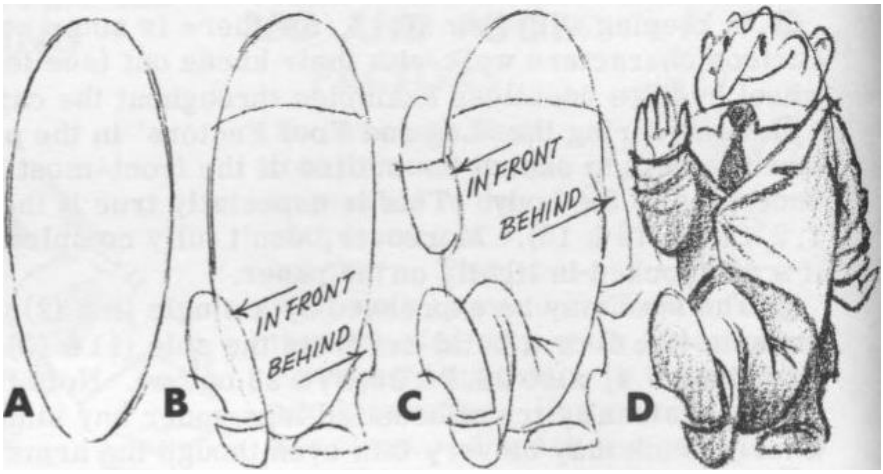


6. Detexturing as optimization

Because the rebalancing transform wants to make a drawing as close as possible to another, it is optimisation. This optimisation is consistent with Frédo Durand's view (§9) that the artist has specific goals and depiction produces a picture that best satisfies them (Durand 2002 §4). As Aaron Hertzmann says (2010 §4.2), this allows us to think about what we want to compute while largely abstracting away the steps required to compute it. Like the compensatory triangles and the idea of generalised inverse, it aids thinking by letting us ignore detail.

7. Compensating by adding rather than deleting cues: representing the form

Artists add cues and delete, e.g., when depicting three dimensions. Here is an example from Jack Hamm's *Cartooning the Head and Figure* (1967, 60):



Perhaps Hamm draws so many creases because it gives more lines to occlude, thus making the most of a vital clue:

> There are only four ways an artist can produce the illusion of forward motion in two dimensions: 1. By perspective (things getting larger as they come forward, smaller as they go back), 2. By overlap (one thing in front of another), 3. By values (dark and light) and 4. By colour (its several attributes).
>
> The cartoonist must use the first two almost exclusively. When one or more cartoon characters are considered apart from their surroundings, overlap assumes priority over all other ways. For 'Mr. Dumpy' above, the foot overlaps the lower leg, the lower leg overlaps the upper leg, the whole leg overlaps the body, and the body overlaps the remaining foot in the rear. NOT ONLY IN THE FRONT VIEW WALK, BUT IN ALL CARTOON ACTIVITY, THE FOREGOING IS MOST SIGNIFICANT.

Hamm uses occlusion to vivify forward motion. Below is a different aspect of three-dimensionality: cylindrical form:

The drawing is by Len Doust, author of a popular series of how-to-draw books in the 1930s and 1940s. In *A Manual on Sketching from Life*, Doust (1949 plate 12) writes:

> The next essential fact always to have at the back of your mind is "form," or the fact that the figure which you are drawing has thickness as well as outline.
>
> If you study closely the great masters of figure drawing in outline, you will be amazed to discover that they manage to indicate the "form" of a body without the use of shading. How is this done? The secret of these clever drawings is often in certain lines on the figure or head and not actually on the outline—a fold, a collar, a cuff, a crease. Look at these lines carefully, and you will observe that they are very correctly drawn, sometimes even more than the actual outline. A simple illustration of this point is in Fig. E, Plate 12.

8. Pictorial techniques

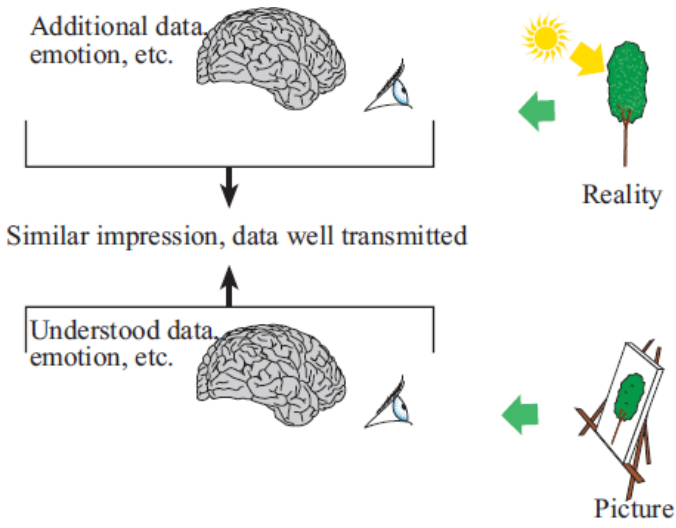
All the above are what Frédo Durand calls "pictorial techniques." This expression is a valuable coinage, which I shall adopt. Durand (2002 §4) explains that pictures are flat, often static, and of restricted extent, contrast, and gamut. Therefore, a direct recording of the scene—such as a photo—may not give the most accurate impression of the original. Some impressions may need to be strengthened, e.g., via the techniques prescribed by Hamm and Doust:

> An image where the contrast at the occluding contour is reinforced might provide a more faithful depth impression, because this compensates for the lack of stereovision or accommodation cues. This is an example of *pictorial techniques* to compensate for the limitation of the medium. A missing cue is rendered through a different perceptual channel (here, stereovision is compensated through occlusion).

Durand's "direct recording of the scene" plays the same role as my "simplistic" or "naïve" drawing. Both are produced from the scene by simple formulaic processes. Both lack or misstate information that the scene contains, so they do not evoke the same impression as the scene in the viewer. Both, therefore, must be repaired with "pictorial techniques":

> Indeed, representing a given scene consists in producing a picture that induces a similar impression to beholders as they would have in front of the real scene (Fig. 6).

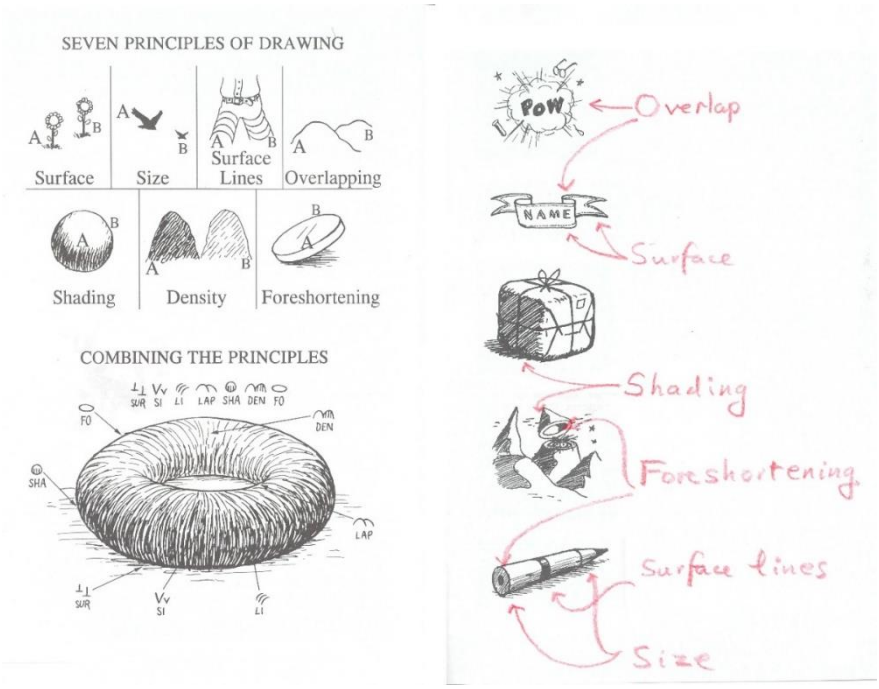
Below is Durand's Figure 6:



The two brains correspond to the two right-hand points in my diagram at the end of §6. The tree and easel correspond to its compensatory triangle's top-left and bottom-right points.

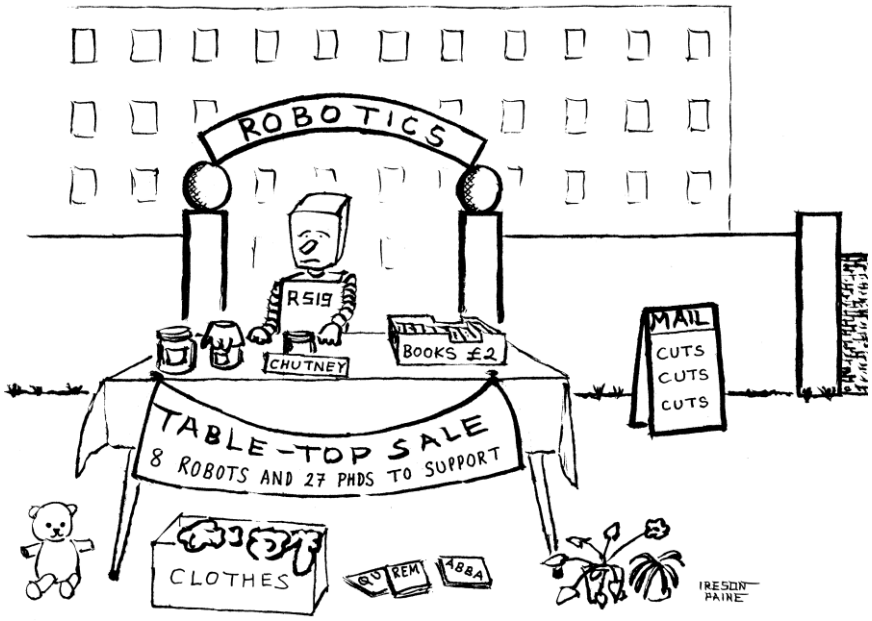
9. Explaining pictorial techniques helps to teach

Teaching these techniques should help novices learn to draw. It will not teach geometrically precise perspective, but it will make drawings three-dimensional, thus more fun and easier to "read." One author to promote this is Bruce McIntyre. The image shows several examples from his *Drawing Textbook* (McIntyre 1998):



10. Non-geometric pictorial techniques: textual anchoring

Added cues need not concern geometry. Thus we can do what Roland Barthes called "anchoring" or "anchorage" (Ludwig n.d.): adding text to convey information that the depiction cannot. So, in my cartoon (Ireson-Paine 2010b), the newsboard saying "CUTS CUTS CUTS" shows the robot seeks funding:

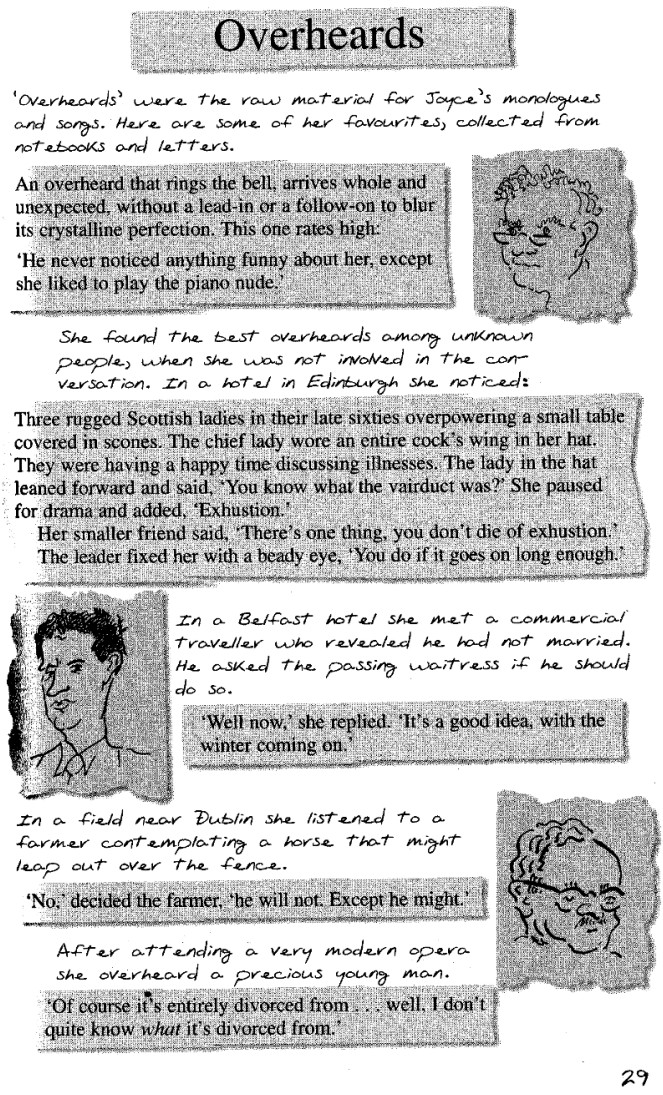


Other uses there and below (Ireson-Paine 2010b) include the labels on the goods, the bee's thought bubble—a pun on spammers' "Make Money Fast"—and the verbs describing its actions:

Textual anchoring may supplement images that are deliberately distorted. Some critics say that Braque and others included text in their paintings: to anchor the picture in reality, despite distortions. The picture below is Braque's *The Portuguese* (The Portuguese n.d.):

11. Distortion and exaggeration as pictorial techniques

Artists also distort to emphasise geometry, as in Joyce Grenfell's second caricature below (Hampton 2004):



Overheards

'Overheards' were the raw material for Joyce's monologues and songs. Here are some of her favourites, collected from notebooks and letters.

An overheard that rings the bell, arrives whole and unexpected, without a lead-in or a follow-on to blur its crystalline perfection. This one rates high:

'He never noticed anything funny about her, except she liked to play the piano nude.'

She found the best overheards among unknown people, when she was not involved in the conversation. In a hotel in Edinburgh she noticed:

Three rugged Scottish ladies in their late sixties overpowering a small table covered in scones. The chief lady wore an entire cock's wing in her hat. They were having a happy time discussing illnesses. The lady in the hat leaned forward and said, 'You know what the vairduct was?' She paused for drama and added, 'Exhustion.'

Her smaller friend said, 'There's one thing, you don't die of exhustion.'

The leader fixed her with a beady eye, 'You do if it goes on long enough.'

In a Belfast hotel she met a commercial traveller who revealed he had not married. He asked the passing waitress if he should do so.

'Well now,' she replied. 'It's a good idea, with the winter coming on.'

In a field near Dublin she listened to a farmer contemplating a horse that might leap out over the fence.

'No,' decided the farmer, 'he will not. Except he might.'

After attending a very modern opera she overheard a precious young man.

'Of course it's entirely divorced from . . . well, I don't quite know *what* it's divorced from.'

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Notice the chin: a bent U which occludes the left-hand side of his neck. I suspect not all the U was visible and that Grenfell lengthened what was. She might also have pushed the U leftwards to overlap the neck, making occlusion possible.

Quentin Blake exaggerates differently. His graphic language emphasises facial expression, posture, and gesture, including hand positions (The Rose Gallery n.d.):

12. Exaggeration is no deceit

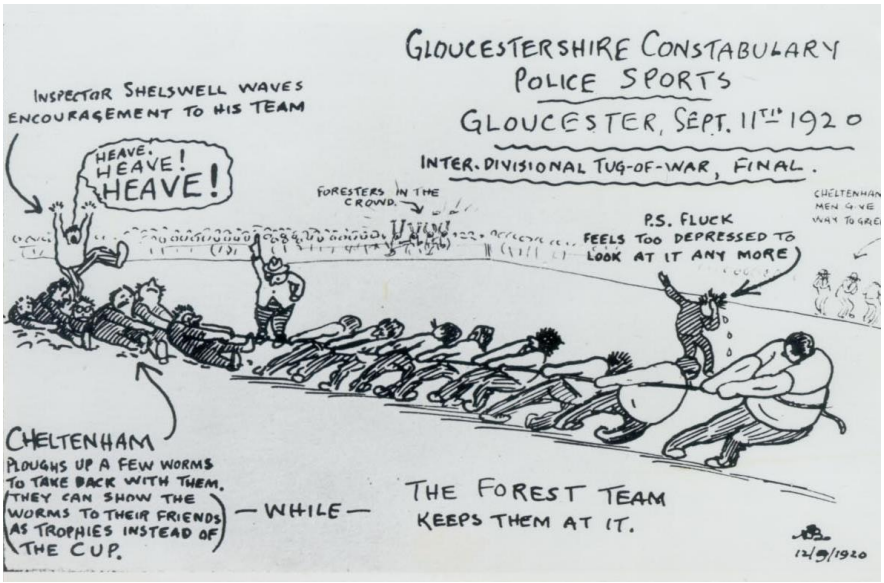
We can argue that the exaggerations above are not "deceit" or "lying," any more than deleting texture or adding lines for form are dishonest. In one way, they push a drawing or painting further from its original scene. However, in another way, they bring it closer by making its effect on the viewer closer to the original scene.

Mathematician Paul Halmos says the same about language. Discussing communication in his autobiography, he writes (Halmos 1985 p. 113) that you may lie a little to make your message clear but should never mislead. For example, consider explaining English governance to a Martian. Saying "England is a monarchy," tells the truth but misleads by implying things that are not true. In contrast, "England is a democracy" is a lie. Nevertheless, it is a better first-sentence summary than the other sentence. It says what is needed as closely as possible, given the listener's lack of knowledge and the available space.

So exaggeration and other pictorial techniques are not lies. They say what must be said as closely as possible, given the constraints of the language.

13. Inflating significant zones

Like detexturing and adding depth cues, inflating significant zones (Ireson-Paine 2019; Hofstadter 1985, 597) compensates for change of graphic language: in this case, loss of detail or resolution. The artist enhances zones critical for recognition, probably at the expense of those not enhanced. Thus below (Gloucestershire Police Archives n.d.), the upraised arms of the "Foresters in the crowd" are emphasised relative to the rest of the body:



14. Compensating for the medium: clouds and blond hair

It is challenging to draw clouds in black ink on white paper. As Len Doust remarks in *A Manual on Sketching Sea, Town and Country* (Doust 1950a, 43):

> When making a simple line drawing, it is usually wisest to leave the sky untouched. You will often be tempted to put in a beautiful cloud formation, but if you do so, you will, nine times out of ten, lose the softness, overdo the tone, and get an unpleasant hardness and solidity; the cloud will jump out of the picture, and your harmony will be lost.

Similar care is needed with blond hair, as in this *Punch* cartoon by J.W. Taylor (Lynch 2008). He reduces the boy's hair to an outline and a hint of locks, compensating for the difficulty of drawing it in detail:

"And don't say 'Oh, what a noble mind is here o'erthrown' every time your father opens his mouth, or I'll stop you going to the pictures altogether."

15. Style as a collection of compensatory triangles

Generalising, I suggest that an artist's style be specified as a collection of compensatory triangles, each describing a particular pictorial technique.

Each triangle can also indicate how extreme its technique is. How much texture does an artist remove? How many creases do they draw? As the pictures below reveal (Chantelle's Blog 2012; Friends NYC 2020), Dr Seuss is extreme regarding creases and wrinkles, and it makes his drawings obscurely unsettling and "boneless," especially as creases appear in skin and fur as well as fabric.

16. Style can be a conscious choice

Some books on drawing advise you not to force a style for yourself; it will gradually come as you learn. That may be partly true because drawing is a physical activity and entails learning neuromuscular habits.

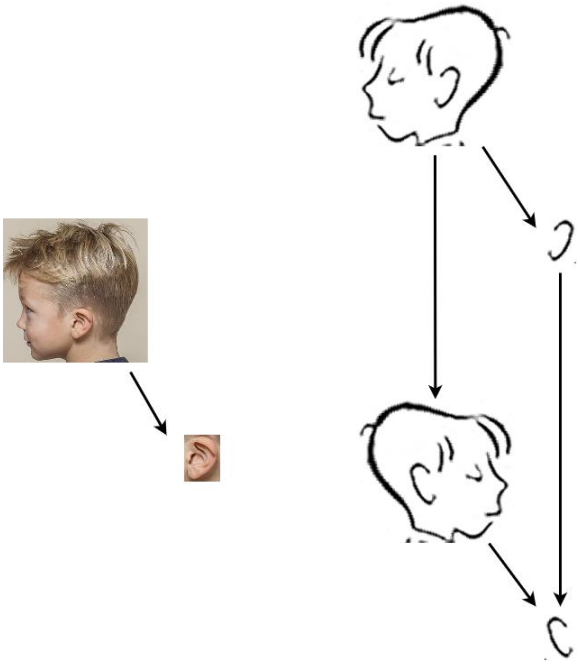
However, style is also about choosing a medium and a graphic language to use with it. Any graphic language will make some things easy to represent and others hard. The hard things can be compensated for by using pictorial techniques. There are trade-offs for each style, and these can be consciously compared. So to that extent, one need not let style happen. One can choose it, analysing the choice as a problem in optimisation.

17. The medium wrecks the message: non-transportability between styles

As noted in §4 and §15, pictorial techniques may create ambiguities or make it harder to depict certain things. This ambiguity leads to my second formulation of style. If a relation can be depicted by one style, how well does it

"transport" to the other? The way I express this may seem to come out of nowhere, but a standard construction in category theory inspires my expression, the "natural transformation." Mathematically analogous examples appear in (Phillips 2021, §2.1; Tsuchiya & Saigo 2021, Figure 5), while (Mannone & Luca 2019, §3) use an identical construction for comparing dance styles.

Taylor cartoon heads as shown in §15. So imagine two cartoonists: J.W. Taylor and W. J. Rolyat. Rolyat does the same, except that he reverses everything he draws. Despite this difference, Rolyat's style precisely mirrors Taylor's:



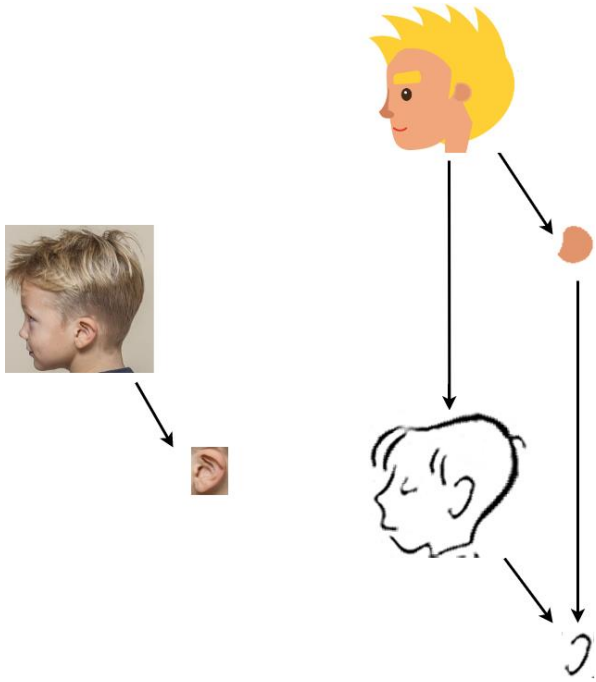
The two-item network on the left above is a schema (Milewski 2015) which describes parts of a scene and how they are related. For simplicity, I consider only one part, the ear. The scene is a blond-haired boy's head (Blond boy n.d.). The diagonal arrow symbolises the relation "part of," i.e., the ear is part of the head.

On the right are two more networks. They work similarly but describe the same head as cartooned by Taylor and Rolyat.

Now consider the parallelogram of arrows. The downward arrows map the items from one style to the other. In other words, they restyle from Taylor to Rolyat. The entire parallelogram says we can get from Taylor's head to the Rolyat ear in two ways. We can select the ear from the original, then restyle it. Alternatively, we can restyle the entire head, then select the ear from the translation.

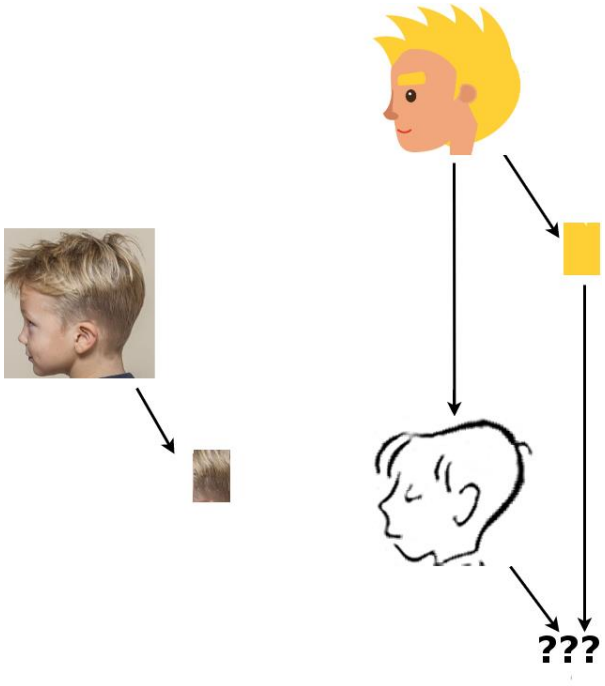
This function is the category-theoretic way of saying that translating between styles does not interfere with the content; i.e., style and content are independent (Libeako 2018). Intuitions on why it means this can be gained from (Ireson-Paine 2021b, Chapter 14), where I describe noughts-and-crosses and two equivalent games. The equivalences mean that moves, pieces, rules, etc., can be translated from one game to another at any point during play without interfering with a game's progress, i.e., a game's "style" is independent of its "content." So both paths do the same thing.

Now consider this diagram:



The first style is now that of a piece of clip-art, modified from (Blond boy 2, n.d.). The second style is Taylor's. As before, both paths around the parallelogram do the same thing, so the style is independent of the content.

However, now consider the hair above the ear:



No item in the second style corresponds to the patch of yellow hair in the first. So Taylor's style has interfered with the content.

Interestingly, this differs from the situation in computing. Computer scientists talk about different data representation methods, e.g., different layouts of tables in a `database`. We can usually `translate` completely between these representations. However, in art, we cannot: medium interferes with message. (However, computer scientists might note that this is not always true of so-called "neural networks." It might be fruitful to apply these ideas to them.)

18. Why mathematise style?

The word "style" gets used loosely in art. Mathematics is the most precise language that we have. By formulating style mathematically, we could make clear precisely what it is.

It would also help us write computer programs that restyle artworks. Today's most popular restyling technique, "deep-learning style transfer," reinvigorated the field after its seminal paper was published in 2015 (Gatys, Ecker & Bethge 2015). One iconic result is pictures of Thurbingen in Germany, restyled to match famous paintings (Mence 2016):

Impressive as this is, it can fail, as shown below. This default is because the programs lack high-level knowledge about style, knowing only statistical summaries of the spatial distribution of texture and pattern. Category-theoretic formulations might give them that knowledge.

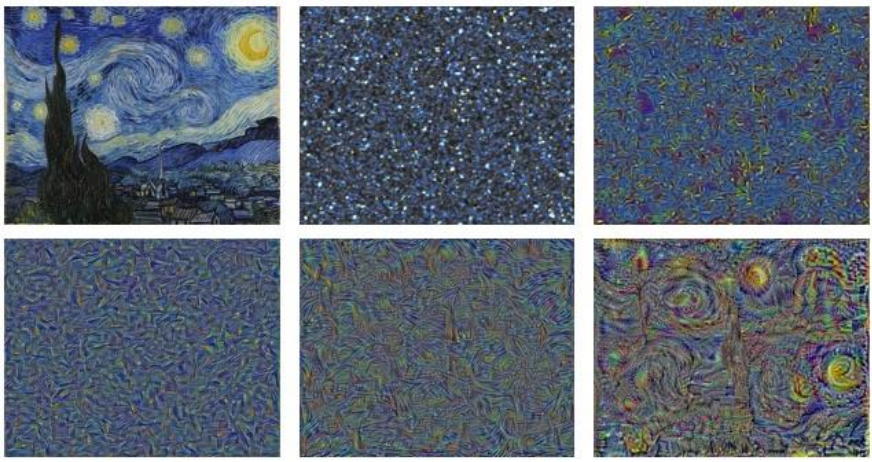
19. When deep-learning style transfer fails

I demonstrated this using the DeepArt website (Gatys et al. n.d.) to restyle photographs of the National Technical University of Athens in the style of the Athenian artist Alekos Fassianos (Ireson-Paine 2019):

The NTUA photos are in the middle, the Fassianos styling images on the right, and the results on the left. It is clear that DeepArt does not understand how to transfer from Fassianos's stylised characters to the real people in the photos. It has not even done an excellent job on the left wing of the NTUA building, putting red stipple from Fassianos's foreground into the wall.

20. Cubism: a test case for theories of style

The above experiment shows that this type of style transfer can fail, and I shall now argue that it will fail when asked to restyle to one style, Cubism. DeepArt represents style as a spatial distribution of patterns at different levels of detail. Mence (2016) demonstrates this by "painting" the levels of style detail it extracts from van Gogh's *The Starry Night* onto blank canvases:

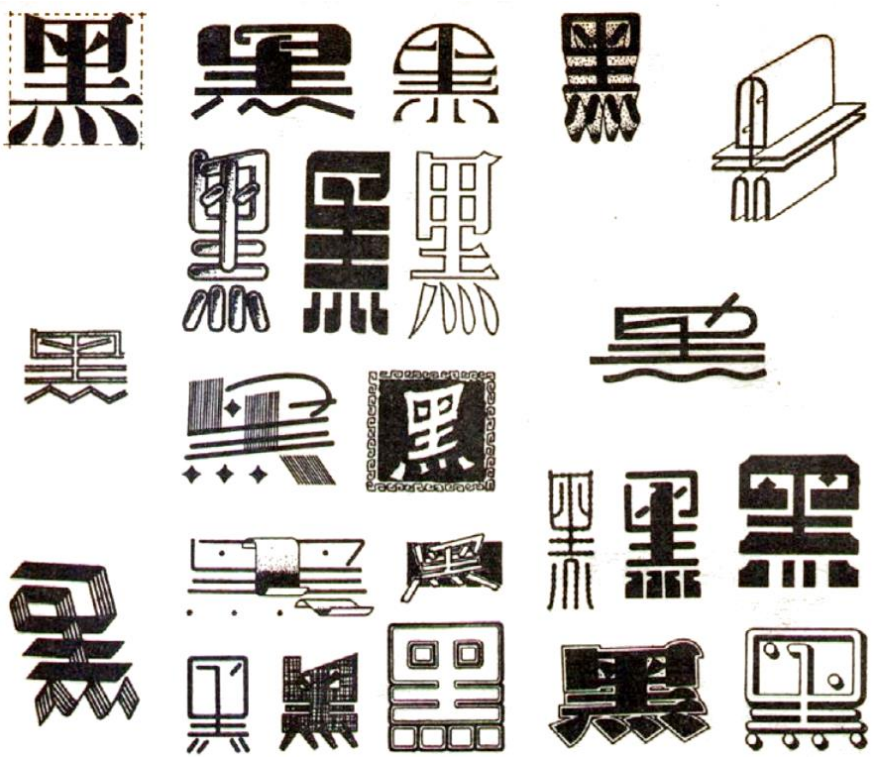


Similarly, the second and third panels below depict style—textures and patterns—learned by the Fritz AI (2021) style-transfer software.

However, Cubist-ising involves more than adding patterns. Thus (Vergo 1980) writes of Cubists using a "mobile perspective": merging different views of the same object even when these were of regions far apart. Likewise, (Gompertz 2012) indicates the importance of multiple viewpoints by noting that Braque and Picasso painted with muted colours to blend them easily. Moreover, for (Hughes 1991), the Cubists see the world as "a network of fleeting events" or "a report on multiple meanings, on process"; a world "set forth as a field of shifting relationships that include the onlooker." It seems very unlikely that DeepArt and related programs could learn to shift and recombine viewpoints in this way. For one thing, there probably is not enough information in the single-viewpoint original image.

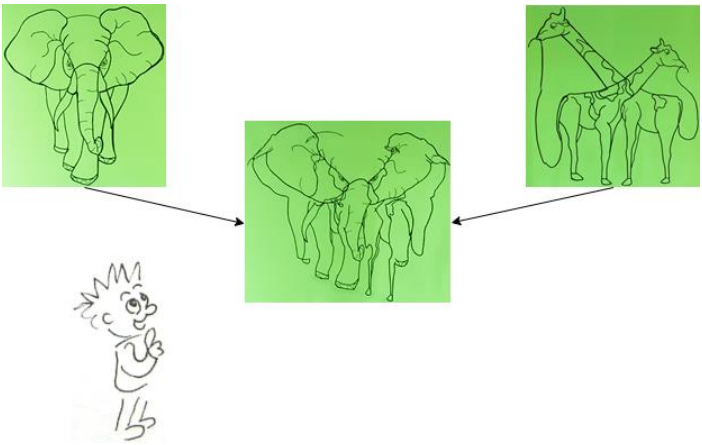
21. Other formulations of style

If Cubism is one test case, typefaces are another. Their simplicity strips the problem of style to its fundamentals. What, for example, unites these renderings of the Chinese character 黑 (Hofstadter 1985 p. 244)?



Such questions arise throughout art and again confirm the need for precise formulations. So how might we go beyond those above?

One way could be to make styles themselves the nodes of a network. We would then study the links between these, asking, for example, which style transfers are invertible. We can restyle Thurbingen as painted by van Gogh, but can we recover the Thurbingen from the van Gogh?



However, how can styles be nodes when they are not "things"? First, category theory lets entire networks be nodes in other networks. So if we can represent styles as networks of relationships, that is a plausible starting point.

Second, style-transfer programs represent style, as shown in the previous section. We could try using these representations as nodes. As I have shown, the programs have defects but might still be good starting points. Promising here is (Chen et al. 2017), a new interpretation of style transfer designed to represent styles explicitly and be easy to analyse.

The explicit representation enables styles to be merged across an entire artwork or in specific regions. This merging is interesting because category theory has an operation called "colimit": a generalised sum that describes how systems such as computer programs acquire function from their parts (Goguen 1992 §3.3). Formulating style fusion as colimit would give us experience by applying category theory to a small and well-defined aspect of style.

Third, we do not need to know what nodes "are," but only how they relate to other nodes. Taking this to its ultimate, we can invoke the "Yoneda perspective" (Bradley 2017), representing an item by its interactions with all the other items. An item is, in a sense, no more than its interactions.

Bradley illustrates a sculpture with a video that, from one angle, looks like a giraffe but from another, an elephant. We can only understand it if we view it from all possible vantage points. This mind-expanding idea forces us to see phenomena not in isolation but as essential components of a more expansive universe.

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