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DO NOT LET THE DEAD BITE! DIFFERENT SCENARIOS OF THE ZOMBIE EPIDEMIC REEXAMINED

ABSTRACT

The Zombie Epidemic is a fun framework for investigating different scenarios of spreading disease. An extended *Kermack – McKendrick* model is analyzed. The only thing that can save humanity is to not get bitten or to find a remedy for the ”zombie virus” (both almost impossible).

KEYWORDS

Zombie, epidemic, Kermack – McKendrick model, ordinary differential equations

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INTRODUCTION

Zombies are fictional undead creatures (Figure 1), brought to life by magic[1] or by a virus.[2] Commonly found in horror and fantasy genre works,[3] zombies have been regarded as mindless, reanimated human corpses feeding on human flesh.[4] However, originally zombies are deeply rooted in tribal beliefs and rituals.[5]

Figure 1. Zombie from the TV series "The Walking Dead"

Source: http://amc.com/shows/the-walking-dead [accessed: 8.12.2015].

[1] W. Booth, *Voodoo Science*, "Science" 1988, No. 240, p. 274.

[2] D. Crisite, S. J. Lauro, *Better Off Dead: The Evolution of the Zombie as Post-Human*, New York 2011, p. 169.

[3] *List of Zombies Films*, [Online] https://en.wikipedia.org/wiki/List_of_zombie_films [accessed: 8.12.2015].

[4] M. Mogk, *Everything You Ever Wanted to Know About Zombies*, New York 2011, p. 214.

[5] E. B. Edmonds, M. A. Gonzalez, *Caribbean Religious History: An Introduction*, New York 2010, p. 111.

Recently the zombie subject has started to blossom in bio-mathematical science as a fairly good framework for simulating the spread of disease.[6] Nonetheless, the main question stayed the same: can we withstand a zombie apocalypse?[7] What can happen and what shall people do to survive? In this work four possible scenarios based on the Kermack–McKendrick model[8] will be described.

MODELS

For the purpose of this article we will be considering six distinct classes:

Susceptibles (*S*) – human population,

Infected (*I*) – those who carry a "zombie virus",

Zombies (*Z*) – those who lost an encounter with a zombie or were resurrected,

Quarantined (*Q*) – those under hospitalization,

Removed (*R*) – those who died after an encounter with a zombie or permanently killed zombies (decapitation),

Unaffected (*U*) – those who are immune to the "zombie virus" forever.

A relatively short timescale[9] will be taken (up to 200 units of time regarded as days). Birth and background death rates are ignored.

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[6] D. Calderhead, M. Girolami, D. J. Higham, *Is It Safe to Go Out Yet? Statistical Inference in a Zombie Outbreak Model*, "Department of Mathematics and Statistics Research Report" 2010, No. 6, p. 1; P. Munz, I. Hudea, J. Imad, R. J. Smith, *When Zombies Attack!: Mathematical Modelling of An Outbreak of Zombie Infection*, [in:] *Infectious Disease Modelling Research Progress*, eds. J. M. Tchuenche, C. Chiyaka, Ottawa 2011; M. Crossley, M. Amos, *SimZombie: A Case-Study in Agent-Based Simulation Construction*, "Agent and Multi-Agent Systems: Technologies and Applications Lecture Notes in Computer Science" 2011, No. 6682, p. 514; R. Smith?, *Mathematical Modelling of Zombies*, Ottawa 2013; A. A. Alemi, M. Bierbaum, C. R. Myers, J. P. Sethna, *You Can Run, You Can Hide: The Epidemiology and Statistical Mechanics of Zombies*, "Physical Review E" 2015, No. 92, p. 052801, DOI: PhyRevE.92.052801.

[7] T. C. Smith, *Zombie Infections: Epidemiology, Treatment, and Prevention*, "The British Medical Journal" 2015, No. 351:h6423, DOI: 10.1136/bmj.h6423.

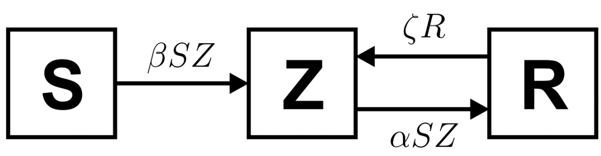
[8] W. O. Kermack, A. G. McKendrick, *A Contribution to the Mathematical Theory of Epidemics*, "Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences" 1927, No. 115 (772), p. 700.

[9] Values of (S), (I), (Z), (Q), (R) and (U) are time dependent and their derivative over time will be marked by a dotted symbol, i.e. $\dot{S} \equiv dS/dt$.

SZR

The SZR model (Figure 2) is one of the simplest ones. Susceptibles, can become zombies through transmission via an encounter with a zombie (transmission parameter β) and by being resurrected from the removed class (parameter ζ). Zombies can be moved to the removed class only by decapitation (parameter α).

Figure 2. SZR model



Source: Own work.

In overall, this model is described by three ordinary differential equations (ODEs):[10]

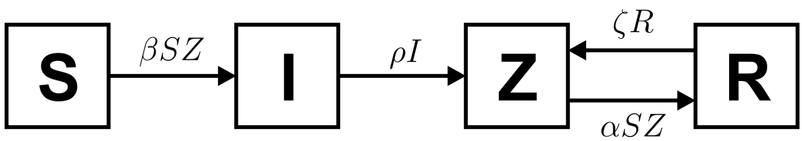
$$\begin{aligned} \dot{S} &= -\beta SZ, \\ \dot{Z} &= \beta SZ + \zeta R - \alpha SZ, \\ \dot{R} &= \alpha SZ - \zeta R. \end{aligned} \tag{1}$$

[10] G. Teschl, *Ordinary Differential Equations and Dynamical Systems*, "Graduate Studies in Mathematics" 2012, No. 140.

SIZR

The SIZR model (Figure 3) is an extension of the SZR model. In contradiction to the previous model, here we adopt a ”more realistic” scenario of a zombie outbreak where susceptibles are initially infected with a ”zombie virus” (latent infection which lasts 24 hours). In this scheme susceptibles move to an infected class and remain there for some period of time before becoming a zombie.

Figure 3. SIZR model



Source: Own work.

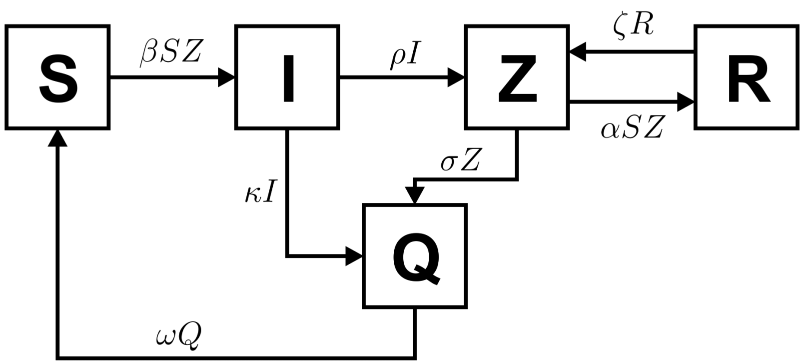
The time evolution is described by the following set of equations:

$$
\begin{aligned}
\dot{S} &= -\beta SZ,\\
\dot{I} &= \beta SZ - pl,\\
\dot{Z} &= pl + \zeta R - \alpha SZ,\\
\dot{R} &= \alpha SZ - \zeta R.
\end{aligned}
\tag{2}
$$

SIZRQ

In this model we add a quarantine class which contains members of the infected or zombie population (entering at rates κ and σ, respectively). Those members are bound to be hospitalized under the threat of being eradicated if something goes wrong. When a quarantined patient is cured, he or she once again becomes susceptible (parameter ω). The scheme is presented in Figure 4.

Figure 4. SIZRQ model



Source: Own work.

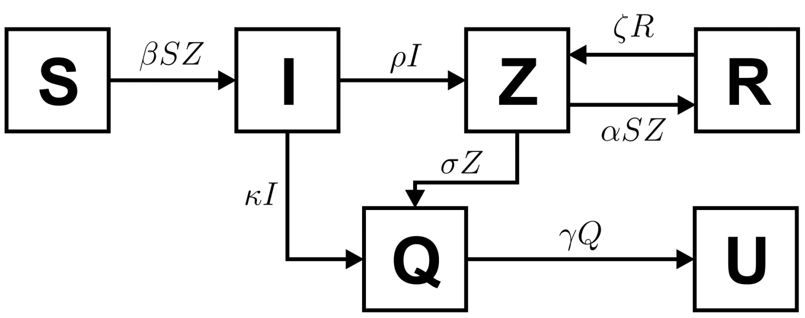
The equations describing SIZRQ model are:

$$\begin{aligned}
\dot{S} &= -\beta SZ + \omega Q, \\
\dot{I} &= \beta SZ - \rho I - \kappa I, \\
\dot{Z} &= \rho I + \zeta R - \alpha SZ - \sigma Z, \\
\dot{R} &= \alpha SZ - \zeta R, \\
\dot{Q} &= \kappa I + \sigma Z - \omega Q.
\end{aligned} \tag{3}$$

SIZRQU

In contradiction to the SIZRQ model, when a quarantined patient is cured, he or she becomes immune to the ”zombie virus” forever (parameter γ). This can only happen after being exposed to contact with a zombie. Preventative vaccination will not make someone permanently immune. The scheme is presented in Figure 5.

Figure 5. SIZRQU model



Source: Own work.

The equations describing SIZRQU model are:

$$\begin{aligned}\dot{S} &= -\beta SZ,\\ \dot{I} &= \beta SZ - \rho I - \kappa I,\\ \dot{Z} &= \rho I + \zeta R - \alpha SZ - \sigma Z,\\ \dot{R} &= \alpha SZ - \zeta R,\\ \dot{Q} &= \kappa I + \sigma Z - \gamma Q,\\ \dot{U} &= \gamma Q.\end{aligned} \tag{4}$$

RESULTS

Table 1 shows the values of the used parameters. The initial population of susceptibles was set to 1000, zombies to 1 and the rest to 0 (see section Models). Numerical calculations were done in *Mathematica* 10.3[11].

Table 1. Parameters and their values for adopted "toy" models

Parameter	Value
α	0.007
β	0.008
γ	0.3
ζ	0.8
κ	0.05
ρ	0.7
σ	0.1
ω	0.45

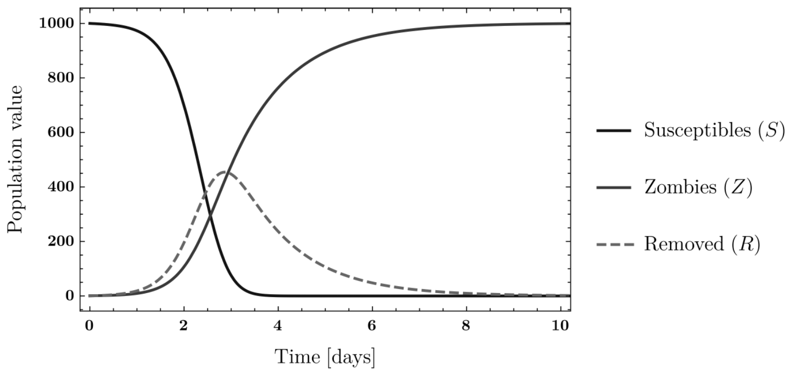
As one can see, a state with only an outgoing arrow will always have population 0 at $t \to \infty$. Furthermore, a state with only an ingoing arrow is an absorbing state (like U in Figure 5).

SZR

The Zombie class overtakes the Removed after three days. The entire human population is eradicated (Figure 6) within four days.

[11] Wolfram Research, Inc., *Mathematica*, version 10.3, Champaign, IL (2015).

Figure 6. Numerical results for SZR model

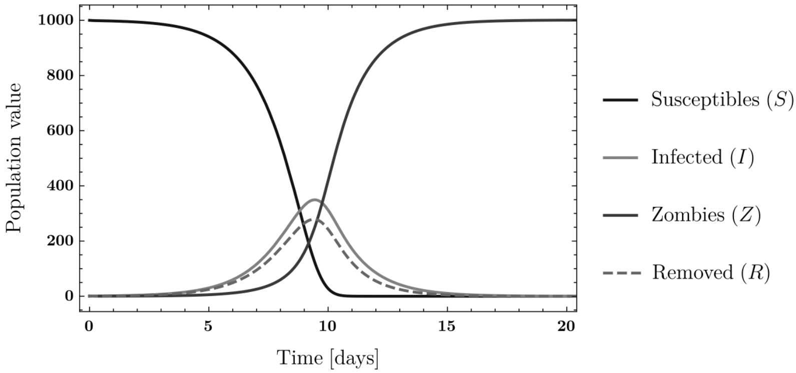


Source: Own work

SIZR

The Zombie apocalypse occurs after ten days. The Infected and Removed class survive up to the fifteenth day. Eventually everyone is zombified (Figure 7).

Figure 7. Numerical results for SIZR model

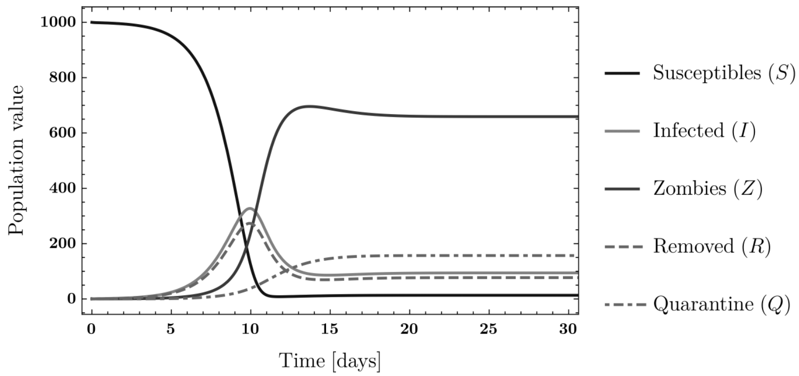


Source: Own work.

SIZRQ

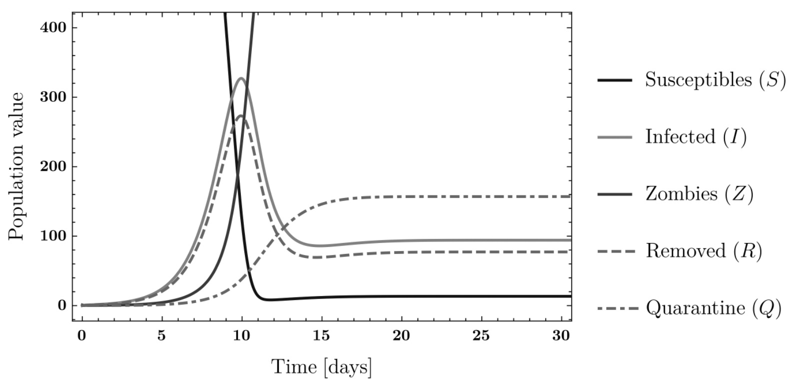
In this scenario the infected and zombified entities are quarantined at a fixed rate. We assume that it is always a successful venture and no one, who is hospitalized, ends in Removed class. However, those who are cured are becoming once again susceptibles. For this model as for the SIZR model the tenth day appears to be critical turning point. After thirty days, equilibrium is established between all classes (Figure 8 and 9). Data are shown in Table 2.

Figure 8. Numerical results for SIZRQ model



Source: Own work.

Figure 9. Magnification of part of Figure 8



Source: Own work.

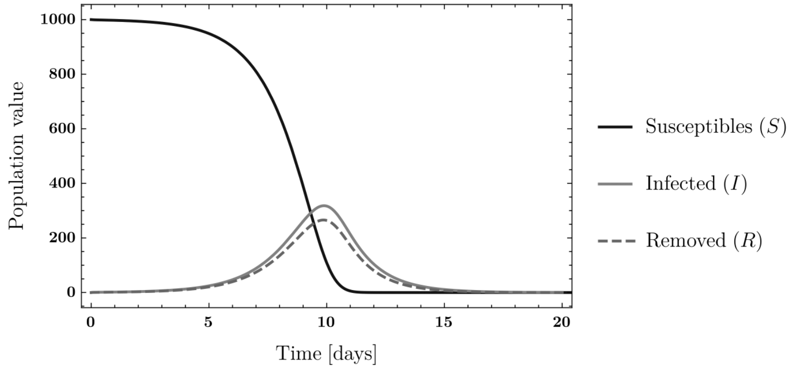
Table 2. Equilibrium state for all classes in SIZRQ model

Class	Population value
Zombies	659
Quarantine	157
Infected	94
Removed	77
Susceptibles	14

SIZRQU

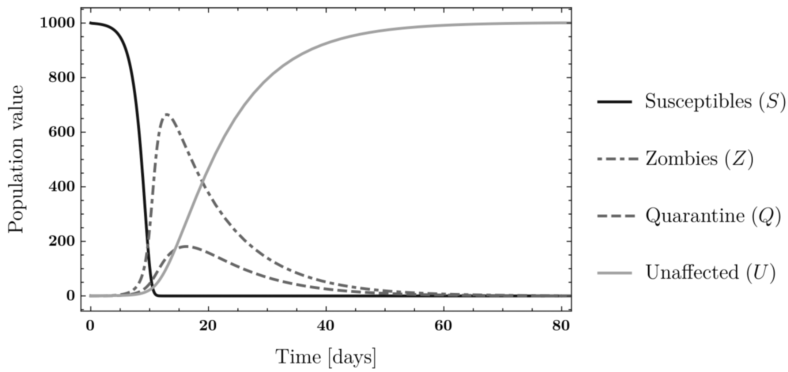
After ten days the population of immune people starts to grow. Henceforth there are no susceptibles and the Infected and Removed class start to decrease. Eighty days are enough to immunize people and get rid of all zombies (Figure 10 and 11).

Figure 10. Numerical results for SIZRQU model – SIR part



Source: Own work.

Figure 11. Numerical results for SIZRQU model – SZQU part



Source: Own work.

CONCLUSIONS

In summary, humans in mostly all presented scenarios are overwhelmed by zombies, except for one where a miraculous vaccine makes people immune to the "zombie virus". That means, that before the zombie outbreak, we have to support the development of vaccinations.

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